DOCUMENT RESUME

ED 200 417 SE 034 541

AUTHOR	Champagne, Audrey B.; And Others
TITLE	Time Out for Problem Solving.
INSTITUTION	Pittsburgh Univ., Pa. Learning Research and Development Center.
SPONS AGENCY	National Inst. of Education (DHEW), Washington, D.C.
REPORT NO	LRDC-1979/22
PUB DATE	79
NOTE	46p.; Not available in hard copy due to small print throughout entire document.
EDRS PRICE	MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS	Cognitive Development; *Cognitive Processes; *Educational Research; Elementary Secondary Education; *Geometric Concepts; Geometry; Learning Theories; Mathematical Concepts; *Mathematics Education; *Mathematics Instruction; *Preschool Education; *Problem Solving
IDENTIFIERS	*Mathematics Education Research

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ED200417

LEARNING RESEARCH AND DEVELOPMENT CENTER

TIME OUT FOR PROBLEM SOLVING

1979/22

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The preparation of this paper was supported by the Learning Research and Development Center, supported in part as a research and development center by funds from the National Institute of Education (NIE), United States Department of Health, Education, and Welfare. The opinions expressed do not necessarily reflect the position or policy of NIE, and no official endorsement should be inferred.

Abstract

Teachers in elementary schools, supervisors of instruction, and other educational practitioners are the primary audience for this publication. The paper presents philosophical, psychological, and practical reasons for including a problem-solving approach in elementary school instruction. It draws on the writings of John Dewey, Jean Piaget, James Greeno, Robert Glaser, Lauren Resnick and others to bring together ideas about problem solving that are useful for the practicing teacher. In addition, the paper contains a detailed illustration of the use of a problem-solving approach to teach elementary school children the important concept of area.

TIME OUT FOR PROBLEM SOLVING

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An elementary school classroom is a busy place. The hours spent there are filled with many activities that occupy the attention of the teacher and the children throughout the whole school day. But have you ever stopped to consider how both the elementary-school teachers and the students might benefit by taking some time out for problem solving?

In this paper, we present some reasons educational theorists give for why problem solving in the classroom is beneficial to students. We show how psychologists study the mental processes associated with learning and reasoning, and we share some of the insights they have gained. We hope to convince you that any time spent on problem solving in the classroom is very worthwhile. And, we intend to demonstrate how a problem-solving method of instruction can be implemented. By a problem-solving method, we mean instruction that includes problem-solving activities or that has as a goal the development of students' skills as problem solvers.

To help you try a problem-solving approach, we have prepared some instructional materials--a module on area--that you can use to take some time out for problem solving in your own classroom. The area module is included in its entirety as an appendix to this paper. It embodies some aspects of a problem-solving method of instruction that we believe will enhance your students' conceptual understanding and their development as learners. As we discuss this method, we refer frequently to various portions of the area module. We think you will find it useful to look at the particular portions of the area module when we refer to them, even though you might not read the

entire module until later. Examples from the module are used to illustrate how some of the things that educational theorists and psychologists have said about the relationship between learning and problem solving can be brought to bear on instruction in the classroom.

Just how, a busy teacher might ask, can I find time in an already overcrowded school day to teach problem solving? Teachers are supposed to be teaching basic skills like reading and arithmetic. Isn't it more important for children to be competent in these basic skills than for us to start worrying about frills like problem solving? What does problem solving mean anyway? Isn't that just another name for arithmetic practice? Or, another teacher might react: Teachers don't teach problem solving, they do problem solving in the classroom every day. If teachers did not solve all kinds of problems that arise, how could they provide meaningful learning experiences for 35 energetic students in a classroom?

We address some of these concerns in this paper and propose that instructional techniques aimed specifically at developing problem-solving skills can improve a teacher's effectiveness. We will explore the views of some familiar theorists--Dewey, Thorndike, Piaget, and others--and suggest how their ideas about problem solving can directly help teachers in their day-to-day work with children. When you have read the whole paper and studied the accompanying instructional module, we think you will agree that teaching problem solving is not only possible but is also an important component of good instruction.

Problem Solving as a Goal of Education

You may have noticed that statements of educational goals inevitably include some demand for problem solving or reasoning, for logical or reflective thinking. The person primarily responsible for this is the philosopher, John Dewey, whose work and ideas had a signal influence on education (see Archambault, 1964). Dewey thought the principal advances of human knowledge and the betterment of the human condition could be credited to the method of science--reflective

thinking or innovative problem solving. Thus, he believed that teaching the method of science should be the primary goal of education.

> Scientific method is not just a method which has been found profitable to pursue in this or that abstruse subject for purely technical reasons. It represents the only method of thinking that has proved fruitful in any subject--that is what we mean when we call it scientific. It is not a peculiar development of thinking for highly specialized ends; it is thinking so far as thought has become conscious of its proper ends and of the equipment indepensible for success in their pursuit. (Dewey, 1910, p. 126)

In his classic book, How We Think (1910/1933), Dewey urged classroom teachers and other educators to stress reflective thinking in the education of children. Dewey's analysis of the stages involved in reflective thinking and his suggestions for promoting reflective thinking in school and classroom practice have influenced generations of educators. Interestingly enough, although Dewey himself did not lay great stress on the stages or phases of reflective thinking, many of his educational followers did. Dewey cautioned in How We Think that his brief summary of phases of reflective thinking (pp. 106-118) did not imply his belief in any particular set or order of phases. Though the listing of five, six, or seven steps of "the scientific method" has sometimes been attributed to Dewey, such lists are really the products of later writers who popularized (and sometimes distorted) Dewey's ideas. Attempts to devise actual instructional methods and materials to develop reflective thinking in students have for the most part been elaborations of Dewey's suggestions and ideas (see Champagne & Klopfer, 1977). These elaborations have been variously labeled scientific thinking, critical thinking, logical reasoning, the method of intelligence, discovery learning, inquiry, and problem solving.

The emphasis placed on problem solving by today's educators can be traced directly to the philosophy of John Dewey. However, commitment to Dewey's philosophy is much more evident in goal statements than in educational practice. There are many reasons contributing to this state of affairs. For one thing, not all teachers agree that teaching students to be problem solvers is one of their functions.

Nor do teachers appreciate how their teaching can be enriched by using problem solving as a method of instruction. This situation is not surprising, since there exist diverse points of view as to what constitutes problem solving. To complicate the picture, it is not easy to grasp the distinction between problem solving as an instructional method and as an outcome of instruction. Further, even teachers who are aware of the benefits of a problem-solving approach to learning may lack the time to prepare suitable problem-solving experiences to use in their classrooms.

The accompanying module on area is a result of our efforts to design problem-solving methods of instruction for the elementary school classroom. Our efforts have been influenced by Dewey, but also by the work of several psychologists. In the remainder of this paper we will describe some pertinent ideas about problem solving from psychological theory and explain how and why these ideas guided the design of our problem-solving module. We hope to persuade teachers of the worth of including problem-solving activities as a regular part of instruction. In our area module, we provide actual instructional materials that may be used to teach children problem-solving skills as they are learning about the concept of area. We intend the area module to stand as an example of the kind of problem-solving instruction teachers would find profitable and enjoyable to use, not only in mathematics, but in various other subject areas.

Problem Solving and Learning: Some Ideas from Psychological Theory

Children constantly confront and solve problems. They engage in problem-solving activities in school, at home, and while they play. They solve social problems (How can I get out of emptying the garbage?), mechanical problems (How do I get the chain back onto the drive gear of my bike?), and conceptual problems (Today my teacher told me I live in Pittsburgh and Pennsylvania and the United States. How is it possible for me to live in three places at once?). Though children can solve on their own many different problems they confront daily, they do need help in learning how to solve the more

complex ones. Thus, teachers and administrators are concerned with a broad range of problems that students must face both in school and in the world outside the school.

In school, most of a teacher's time might be spent providing instruction that will help students learn to solve formal or academic problems. It is this general type of problem that learning psychologists study, and it is also the type we examine in this paper. Before discussing the teaching of skills for solving academic prob[illegible], however, we examine the research and theories of learning psychologists to see what information they provide that we can apply to our goal of bringing more problem solving to the classroom.

We will relate these psychological theories of problem solving to a very practical application in the classroom--teaching the concept of area. Our choice of the area concept as our instructional example was guided by several considerations. First, there is a considerable amount of psychological literature on the concept of area, some of which we describe in this paper. Second, area is a basic concept in mathematics and science, and it is used frequently in various other academic subjects as well. In geography, for example, the concept of area is needed in calculating the amount of arable land a country has to feed its people, and in biology, knowing the surface area an animal has for oxygen-carbon dioxide exchange helps to determine its activity level. Third, the concept of area has many practical applications in people's daily lives. How much cloth is needed to make a skirt, how much linoleum to cover the kitchen floor, how much grass seed to sow the lawn, how much paint for the bathroom walls? All these are questions or problems that most people confront at one time or another. Finally, despite the importance of the idea of area, many children experience difficulty in acquiring an understanding of the concept.

Categorizing Academic Problems: Eight Problem Types

Even within the limited context of formal or academic problems, the question of what constitutes a "problem" exists. Various psychologists have attempted to explicate the characteristics of formal problems

and to suggest ways in which they can be categorized. However, like the blind men describing the elephant, different psychologists and schools of psychology have focused on different aspects of problem solving in their attempts to define a real problem.

Jacob Getzels, a learning psychologist who has long studied problem solving and creativity in children and adults, has provided a convenient framework for defining and categorizing problems and their solutions. Getzels (1964) cites the following scheme, devised by W. L. Libby, for describing types of problems:

Eight Problem Types

1. The problem is given (is known) and there is a standard method for solving it, known to the problem solver (student, experimental subject) and to others (teacher, experimenter) and guaranteeing a solution in a finite number of steps.

2. The problem is given (is known) but no standard method for solving it is known to the problem solver, although known to the others.

3. The problem is given (is known) but no standard method for solving it is known to the problem solver or to the others.

4. The problem itself exists but remains to be identified or discovered (become known) by the problem solver, although known to others.

5. The problem itself exists but remains to be identified or discovered (become known) by the problem solver, and by the others.

6. The problem itself exists but remains to be identified or discovered (as in 4 and 5) and there is a standard method for solving it, once the problem is discovered, known to the problem solver and to the others (as in 1).

7. The problem itself exists but remains to be identified or discovered, and no standard method for solving it is known to the problem solver, although known to the others (as in 2).

8. The problem itself exists but remains to be identified or discovered, and no standard method for solving it is known to the problem solver or to the others (as in 3). (p. 241)

By means of this scheme, Getzels focuses our attention on two attributes of a problem: its origin (Is the problem given or must the problem solver formulate or discover it?), and the status of knowledge about the method of its solutions (Is there a standard method of solution? By whom is the solution known?). All sorts of problems--whether encountered in mathematics, English, social studies, or botany--can be described with respect to these two attributes. By analyzing some of these typical problems in terms of their origin and the problem solver's awareness of solution methods, we will be better able to explain how our problem-solving approach to teaching differs from traditional instruction.

Academic Problems and Problem-Solving Skills

Figure 1 contains examples of typical problems children encounter in primary arithmetic. When these problems are categorized according to the scheme cited by Getzels, they are seen to fall into problem Types 1 and 2. Children confront Type 1 problems when they are given practice problems that can be solved by using a standard procedure (algorithm) they have been taught--like substituting numbers into a formula (for instance, $\underline{A} = \underline{l} \times \underline{w}$ is the algorithm for finding the area of a rectangle) or carrying out long division using the shortcut computational procedure. Type 2 problems involve the application in an unusual circumstance of some algorithm that was previously taught. In these cases, the particular application of a known algorithm has not been pointed out to the child, but it is known to the person presenting the problem.

Getzel's categorization scheme can be viewed as a matrix of problem-solving tasks based on variations in the origins of problems and variations in the extent of knowledge about problem solutions. This matrix is shown in Table 1. The matrix allows us to consider all kinds of problem situations that enable the student to develop or practice certain kinds of problem-solving skills. Having made explicit the kinds of problem-solving tasks to be performed in each situation, we can then describe examples of academic problems that

Type 1 Problems

A field is 30m long and 15m wide.

What is the field's area ?

(application of a substitution algorithm)

147 ÷ 12 = ?

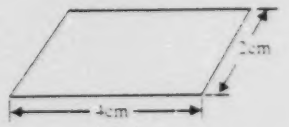
(application of a computation algorithm)

Type 2 Problems

The area of a field is $450m^2$

The length of the field is twice the width. What is the length of the field ?

What is the width of the field ?



What is the area of the parallelogram ?

Figure 1. Problem types 1 and 2. Examples from elementary arithmetic.

Table 1

Problem-Solving Tasks in Problem Types 1-8

Solution Method	Problem Origin: Given or Known to Student and Teacher	Problem Origin: To Be Discovered by Student Known to Teacher	Problem Origin: To Be Discovered by Student and Teacher
Undefined		Type 4 Task: To discover (or formulate) the problem.	Type 5 Task: To formulate the problem.
Standard Method Known to Student and Teacher	Type 1 Task: To use the standard method (or choose some other method) to find "the answer" to the problem.	Type 6 [a] Task: To discover (or formulate) the problem and use the standard method (or some other method) to find an answer to the problem.	Type 6 [b] Task: To formulate the problem and to use the standard method (or choose some other method) to find an answer to the problem.
Standard Method Not Known to Student, But Known to Teacher	Type 2 Task: To discover a method of solving the problem and to find the answer to the problem.	Type 7 [a] Task: To discover (or formulate) the problem, to discover a method of solving the problem, and to find an answer to the problem.	Type 7 [b] Task: To formulate the problem, to discover a method of solving the problem, and to find an answer to the problem.
Standard Method Not Known to Student or Teacher	Type 3 Task: To search for a method of solving the problem. If a method is found, to find an answer to the problem.	Type 8 [a] Task: To discover (or formulate) the problem and to search for a method of solving it. If a method is found, to find an answer to the problem.	Type 8 [b] Task: To seek a way to formulate the problem. If this is done, to search for a method of solving the problem. If a method is found, to find an answer to the problem.

fit the various conditions. Examples of problems for each of the conditions are shown in Table 2.

Using Getzel's scheme to analyze academic problems helps us as educators to make some useful distinctions among problem types. The insights gained from such an analysis help us better to describe current educational practice and to define our goals vis-a-vis incorporating problem solving into children's school experiences. As Getzels himself points out, by applying the problem categorization scheme to arithmetic and other problems children encounter in school, we learn two things about the kinds of problem-solving experiences children are provided there. First, by far the greatest number, if not all, of the formal problems in a child's schooling fall into the lowest level of the possible range of problem types (Type 1). Second, it is easily seen that there is a large range of possible problem types that children encounter infrequently or never in school. In this range of rarely encountered problems lie those academic problems that are most like the ones children and adults must confront and solve in their daily lives.

When we limit a student's instructional experiences to Type 1 problems or to only these and the simplest Type 2 problems, we are not providing students with the opportunity to define problems and discover suitable methods of solution. We are not helping them learn and practice the skills needed to solve the types of problems higher in the classification scheme, skills they will need to solve real-life problems.

Of course, it is clear that certain of the skills needed to solve problems found in the higher levels of the classification scheme are beyond reasonable expectations for young children. Moreover, teaching children to formulate problems, devise innovative solutions, and evaluate their solutions is an ambitious undertaking, one that requires instruction over a long period of time. It seems obvious, too, that we cannot expect children to be successful at formulating and solving problems on their own before they have some successful experience solving problems that are presented to them. The need or opportunity to discover a problem or a solution is more characteristic of real-life

Table 2

Examples of Academic Problems of Types 1-8

Solution Method	Problem Origin: Given or Known to Student and Teacher	To Be Discovered by Student, Known to Teacher	To Be Discovered by Student and Teacher
Undefined		Type 4 The teacher, unbeknownst to the students, cuts the first centimeter from the classroom meter sticks, then asks the students to measure the lengths of some objects. The students' task is to discover that the meter sticks have the first centimeter missing, then to devise a way to measure the objects with the modified meter sticks.	Type 5 For a class lesson on area, the teacher writes various formulas for calculating area on the board, not realizing that one of them is written incorrectly. In the course of the teacher's lesson about these formulas, one of the students finds the area of an object in another way, producing a result in conflict with that obtained by the formula and discovers that the teacher has written an incorrect formula.
Standard Method Known to Student and Teacher	Type 1 The student knows how to calculate the area of a rectangle by the formula $A = l \times w$. (S)he is given the problem: How many square feet of linoleum are needed to cover a kitchen floor 20 feet long by 10 feet wide?	Type 6 [a] A student wishes to build a model of a medieval castle as a classroom project in social studies. The student wants to use a certain kind of fairly expensive art paper to make roofs for the various sections of the castle (e.g., the main hall, turrets, etc.) that will require variously shaped roof pieces (rectangles, cones, etc.), but (s)he does not know how many sheets of paper to buy. In thinking about the problem, the student realizes that the amount of paper can be calculated by figuring out the areas of the pieces needed and mapping them on to the paper sheets.	Type 6 [b] The teacher writes on the board several proofs in geometry, purposefully writing some of the steps of the proof incorrectly. The students are instructed to look the proofs over carefully, and to use the proofs as models to help them devise a proof for a new problem. The students' real task, however, is to discover the incorrect steps in the sample proofs.

--Continued--

Table 2 (Continued)

Solution Method		Problem Origin: Given or Known to Student and Teacher	Problem Origin: To Be Discovered by Student; Known to Teacher	Problem Origin: To Be Discovered by Student and Teacher
	Standard Method Not Known to Student, but Known to Teacher	Type 2 The student knows how to calculate area as above. (S)he is given the problem of finding the area of a specific parallelogram.	Type 7 [a] In the course of a science lesson on changes of state, a student freezes samples of cold and hot water and observes that the hot water becomes solidly frozen before the cold water does. The surprised and curious student wants to "find out why" and is encouraged to do so by the teacher. The student, perhaps gently guided by the teacher to sources of information, plans a series of experiment to investigate this phenomenom and, as a result of the experiments seeks out additional information.	Type 7 [b] The teacher arranges for some Cuisinaire rods to be placed in a corner of the primary classroom. When students ask, the teacher says they may play with the rods if they like, but otherwise, the teacher makes no attempt to interest the students in the equipment, leaving it to students' curiosity to figure out how the sticks may be used.
	Standard Method Not Known to Student or Teacher	Type 3 The teacher comes upon an interesting mathematics problem in a book and shares it with his/her students before knowing how to find the answer. For example, this problem: Smith, Jones, and Robinson are the brakeman, fireman, and engineer of a train, not necessarily respectively. Today only three passengers are riding this train, and . . . their names are . . . Mr. Smith, Mr. Jones, and Mr. Robinson. Here is some other relevant information: (A) Mr. Robinson lives in Detroit. (B) The brakeman lives halfway between Chicago and Detroit. (C) The passenger who lives in Chicago has the same name as the brakeman. (D) The brakeman's next-door neighbor, one of the passengers, earns exactly three times as much as the brakeman. (E) Mr. Jones earns exactly $2,000 a year (and collects a lot of food stamps and welfare payments). (F) Smith beat the fireman at billiards. Who is the engineer?	Type 8 [a] The student, while working on some map-making projects, observes that countries on all maps are differentiated by no more than four colors. The student becomes intrigued by this, and tries to draw a map requiring more than four colors, but is unable to do so. The student happens to read a book on topology and runs across "the four-color map problem." Since the book is a few years old, it discusses the problem as an as yet-unsolved classic problem. The student grows interested in topology and begins reading and studying the subject with an eye to trying to prove the problem.	Type 8 [b] The teacher encourages some students to set up an aquarium in the classroom. Everything works well for some time, then the fish are struck by some mysterious desease. Interested students are encouraged by the teacher to find out the cause of the disease and, if possible, to take the steps needed to prevent it from recurring.

problems than of the academic problems elementary school children are likely to encounter. Yet it is also reasonable to assume that by providing opportunities for children to search for solution methods, as in Type 2 problems, we enable them to develop skills applicable to academic and real-life problem types at higher levels of the scheme.

We have put some of these ideas into practice in our area module, as illustrated by the types of problems selected for inclusion. The area module does contain a number of Type 1 problems, which are intended as opportunities for students to practice previously learned solution algorithms. We are convinced, however, that children benefit from exposure to challenging Type 2 problems that require them to invert methods of solution, and that it is also important for them to confront problems they may have to think about for some time before they are able to devise a solution. Type 2 problems are therefore also included in the module. In a later section, we present some examples of Type 2 problems from the module that persons who understand the concept of area should be able to solve.

Mental Processes in Problem Solving

Describing and classifying problems according to their origins and knowledge about their solutions is a useful process. It provides a basis for analyzing and evaluating contemporary classroom practice with respect to the kinds of problems children encounter in school, and it provides a context in which we can begin to effect change rationally. The theoretical perspectives of learning psychologists also provide other insights that can be profitably employed in designing instruction.

The attention of many learning psychologists is focused not so much on specific characteristics of problems, but rather on the mental processes a person must employ to successfully solve them. There are several different perspectives on the nature of these mental processes and how they are developed. The four perspectives we discuss here are represented in the work of six psychologists: Edward Thorndike, a classical associationist; Max Wertheimer, a Gestalt psychologist; Lauren Resnick, Robert Glaser, and James Greeno, representing

the information-processing perspective; and Jean Piaget, a structuralist. The views of these psychologists have influenced our design of the area module in diverse ways, which we will explain as we go along.

An Associationist View

The perspective of Edward Thorndike, an associationist, is particularly important because we can see his influence in two salient aspects of current classroom practice, namely, the importance of drill, and the value placed on a student's ability to answer questions quickly. Thorndike's theoretical position is that problem solving, like other kinds of thinking, results when we mentally combine or associate ideas or concepts that contain one or more common elements. This viewpoint has led to classroom procedures that rely on frequent practice or drill to strengthen the desirable associations between two concepts. The idea is that if students practice answering factual questions quickly and accurately, they are then able to use this information when presented with a problem to solve.

We believe it is accurate to assert that children in most American schools are taught about area and about a related concept, perimeter, by an instructional approach that is a direct application of classical associationist theory. They memorize a formula, learn how to substitute numbers[1] into the formula, and carry out the necessary calculations. Teaching students to find area and perimeter in this fashion is teaching them to apply an algorithm, or standard method, to a problem situation. This approach is usually highly effective in enabling children to solve problems of the type shown in Figure 2 (except that many students confuse the two formulas and use $\underline{l} \times \underline{w}$ to compute perimeter and $^2\underline{l} + ^2\underline{w}$ to compute area).

The problems shown in the top half of Figure 2 fit perfectly into Getzel's characterization of Type 1 problems: "The problem is given

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[1] We use the term "numbers" rather than the terms "length" and "width" because most children can't distinguish between the length or width of a rectangle but usually label length and width as a function of a rectangle's position on a page.

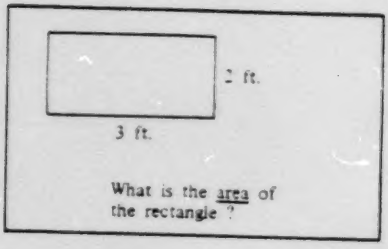
and there is a standard method for solving it, known to the student and the teacher and guaranteeing a solution in a finite number of steps." We expect children who have received the usual instruction on the concepts of area and perimeter to be able to solve them. In fact, in the algorithmic approach to teaching area and perimeter, nearly all of student's work consists of practice with problems like these. One of the inadequacies of this approach is that, after receiving the kind of instruction described above, very few students can solve Type 2 problems like those shown in the bottom half of Figure 2.

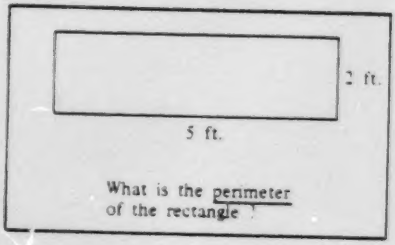
A Gestalt View

Max Wertheimer was a Gestalt psychologist who believed that Thorndike's ideas about teaching mathematics were all wrong. Wertheimer was convinced that teaching by associationist methods fostered "ugly thinking" and a trial-and-error approach to problems, because the emphasis was on memorizing formulas and practicing mechanical substitutions. Wertheimer argued that mathematical concepts should be taught by means of problems that give students a "structural understanding" of the concepts involved. Wertheimer (1945/1959) carried out numerous studies in which he presented people with challenging problems (generally Type 2 problems) that embodied mathematical and geometrical concepts. He analyzed people's approaches to these problems in terms of whether his subjects showed evidence of understanding the underlying concepts or whether they attacked these problems by trial and error. He found that persons whose knowledge of a concept was based on memorized associations were often unable to solve problems presented in non-standard ways and often approached problems in a way that showed they didn't understand the underlying concepts. On the other hand, persons who learned concepts through understanding rather than drill were able to deal successfully with problems (of Type 2) by using "productive thinking" (Wertheimer's term for problem solving).

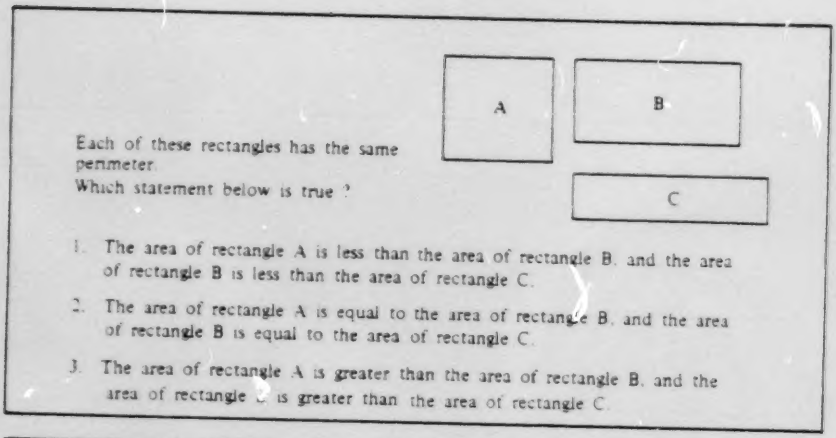
Wertheimer's proposed method for teaching children the concept of area contrasts greatly with the associationist method of memorizing a formula and substituting numbers into it. He argued that the proper

Type 1 Problems





Type 2 Problems


A
B
C
Each of these rectangles has the same perimeter.
Which statement below is true ?
1. The area of rectangle A is less than the area of rectangle B, and the area of rectangle B is less than the area of rectangle C.
2. The area of rectangle A is equal to the area of rectangle B, and the area of rectangle B is equal to the area of rectangle C.
3. The area of rectangle A is greater than the area of rectangle B, and the area of rectangle B is greater than the area of rectangle C.

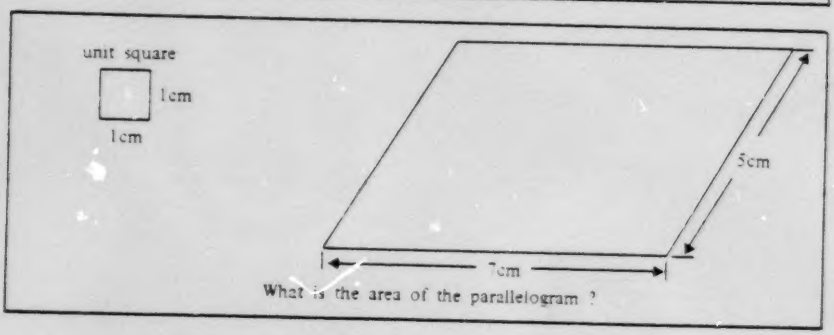


Figure 2. Problems on area and perimeter.

"structural understanding" of area measurement could be developed in students by showing them how to use unit squares to cover an object whose area was to be measured.[2] Children who first learned to measure area in this fashion, according to Wertheimer, would eventually develop a conceptual understanding of the formulas for area measurement. Having this understanding, they would realize, for instance, that multiplying the length of a rectangle by its width gives the same numerical answer as multiplying the number of unit squares in one row by the total number of rows. After children came to understand how to measure the area of a rectangle using unit squares, they would then be given other figures to measure on which the unit squares would not fit evenly. Giving students these opportunities to invent new procedures for measuring area would help them increase their understanding of area measurement and also help them develop strategies for solving problems. Wertheimer was particularly interested in how people who knew how to find the area of a rectangle applied this knowledge when asked to find the area of a parallelogram--a problem that stimulated some of our own interest in designing the area module.

We have used some of Getzels' and Wertheimer's ideas about problem solving in choosing a sequence of problems to interweave with the instructional lessons of the module. Both Getzels and Wertheimer suggest that students' early academic experiences with solving problems should include at least two types of problems: problems that allow students to apply methods they have already learned and problems for which students have not learned a standard method of solution. The content outline of the module is presented in Table 3.

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[2] It is interesting to contrast this assertion with Piaget's description of a child who uses the process to compare the areas of two figures, A and B. Both figures have the same area but different shapes. The child covers each figure and notes that it takes 9 unit squares to cover A and 9 to cover B but still insists that B is bigger. Piaget comments that "perceptual disparity outweighs numerical identity" (1960, p. 279) and then goes on to describe the logical processes that are necessary to correctly interpret the manipulations (covering, counting, and comparing), logical processes that Piaget asserts are not a part of this child's cognitive structure.

Table 3

Content Overview of the Area Module

Lessons	Skills and Content Taught	Associated Problems
Part I. Length	Ordering line segments and sticks by length; measuring lengths of line segments in inches.	
	Practice Sheet I. Length. Comparing lengths of line segments, ordering line segments by length, measuring line segments and distances between points in inches.	
Part II. Squares and Rectangles	Identifying properties of squares and rectangles; identifying right angles, squares, and rectangles; inclusion of the class squares in the class rectangles; identifying and measuring the length and width of rectangles.	Using four lines of equal length to construct an object with four right angles that is not a square.
	Practice Sheet II. Squares and Rectangles. Identifying right angles.	Constructing four-sided objects with specified lengths of sides, kinds of angles, etc.
Part III. Comparing Areas	Observing that congruent rectangles have equal areas, and that rectangles equal in one dimension, but unequal in the other, have different areas.	Determining how many squares will cover a rectangle that is congruent to another rectangle that can be covered by a known number of squares.
Part IV. Measuring Area	Measuring area of a rectangle by constructing a congruent rectangle from unit squares, then counting the squares.	Comparing the relative areas of two rectangles of different shape, one of which has been transformed from a figure congruent to the first rectangle (area conservation). Inventing a method of measuring the area of a narrow rectangle whose width is one-half that of a unit square.
	Practice Sheet IV. Measuring Area. Measuring the areas of rectangle using unit squares.	Inventing a method of measuring the area of a triangle, given that scissors may be used.
Part V. Shape and Area	Comparing areas of rectangles of different shapes whose parts can be rearranged to form congruent rectangles (area conservation). Measuring the area of these rectangles with unit squares.	
	Practice Sheet V. Shape and Area (A). Measuring the area of a figure, using unit squares, before and after transforming its shape. Comparing the areas of the original figure with that of the transformed figure.	
	Practice Sheet V. Shape and Area (B). Measuring the areas of triangles, trapezoids, and other figures by transforming them into rectangles on which congruent figures can be constructed from unit squares.	Inventing a method of measuring the area of a parallelogram.

You will note that each part of the module includes problem-solving experiences of both types. Later, as you read through the module itself, the correspondence between the problem sequence and the lesson sequence will become more apparent.

To illustrate the relation between lessons and problems, observe that Part IV of the module instructs the student in measuring the area of a rectangle by covering the figure with unit squares and counting the squares. A problem at the end of Part IV asks the student to determine the area of a rectangle on which the squares do not fit evenly. The rectangle, shown in Figure 3, is in fact only one-half as wide as the unit squares. A dotted line has been printed on the rectangle and the student is given the hint that scissors may be used in solving the problem. The student's previous experiences with measuring area have all involved rectangles on which the unit squares fit evenly. The student has previously used the scissors in the lessons to cut a figure apart, but has never been explicitly instructed to cut and rearrange parts of a figure if the unit squares do not fit evenly on it. This problem thus provides a challenge to the student's inventiveness without being discouragingly difficult.

Some Information-Processing Views

Another important influence on our work has come from a group of psychologists who view problem solving as information processing. Information-processing psychology is based on the assumption that human beings' mental processes may be productively compared to the operations of computers. The notion behind this view is that human beings, like computers, both store and process information. The series of operating instructions which make up a computer program, and which are stored in the computer's "memory," are seen as analogous to human mental processes which are stored in human memory. Once programmed, the computer can take in information relevant to a specific problem, process the information through the operating instructions of the program, and output a solution to the problem. Similarly, human beings take in information from a problem-solving task, manipulate that information in their minds, and come up with a problem solution.

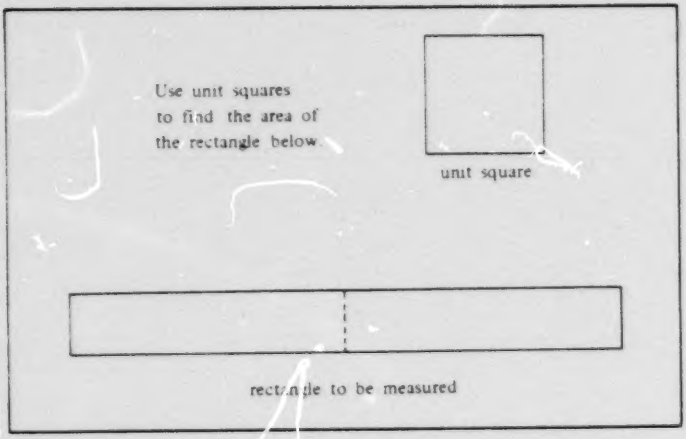
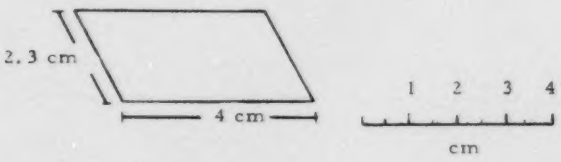


Figure 3. A problem from the area module

To learn more about how humans think, information-processing psychologists try to program computers to solve certain types of problems in the way a human being would. First, they carefully observe the steps humans use to solve a particular problem. Then each of the steps a human problem solver used becomes an operating instruction in the computer program. Writing a computer program helps the psychologist conceptualize and specify the kinds of decisions and the sequence of decisions that humans make in solving particular kinds of problems. The sequence of decisions can be outlined in a schematic diagram, an example of which is shown in Figure 4. Taken from the work of Lauren Resnick and Robert Glaser (1976), this diagram outlines the sequence of decisions used in solving certain types of mathematical problems.

Resnick and Glaser's general scheme is applicable to several interesting types of problems--for example, the parallelogram problem. Imagine a person who knows how to calculate the area of a rectangle using the formula $\underline{A} = \underline{l} \times \underline{w}$. Now imagine that person confronted with the problem of finding the area of a parallelogram, like the following:



Viewing the area-of-a-parallelogram problem in terms of the Resnick and Glaser outline results in the schematic diagram shown in Figure 5. Each rectangle and diamond in the diagram shows where some particular kind of mental processing takes place which moves problem solving along. By following along the arrows in numerical

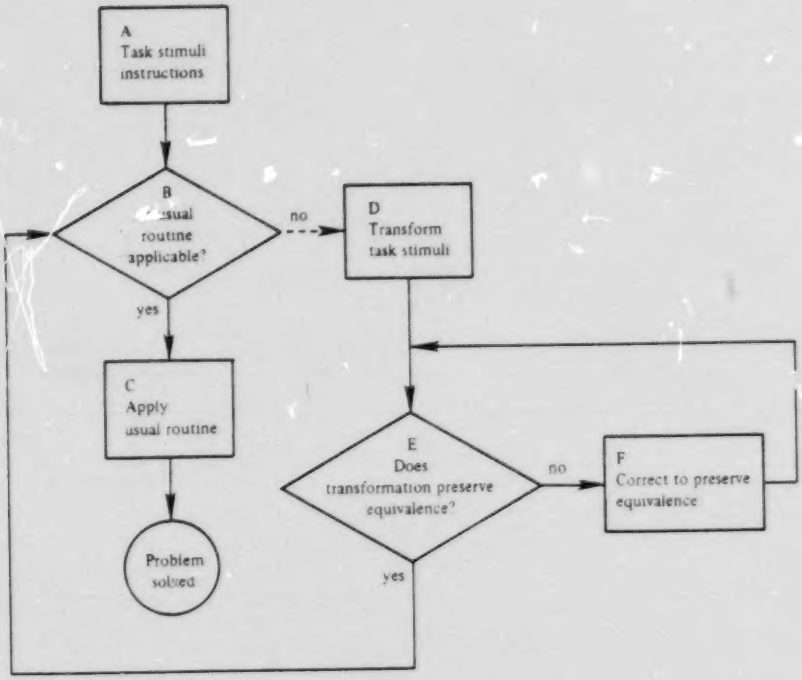


Figure 4. Schematic diagram of successful problem solving. (From "Problem Solving and Intelligence" by Lauren B. Resnick and Robert Glaser. In L.B. Resnick (Ed.), *The Nature of Intelligence*. Hillsdale, NJ: Lawrence Erlbaum Associates, 1976.)

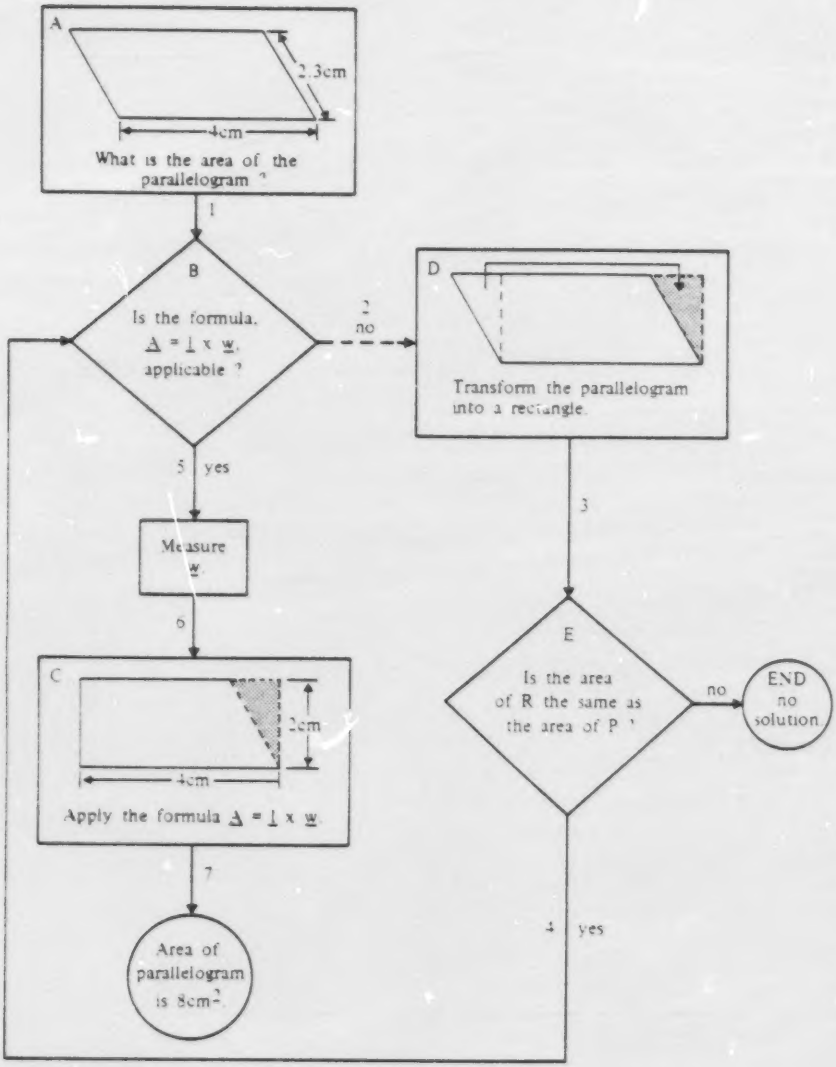


Figure 5. Schematic diagram for solving the parallelogram problem. (Adapted from Resnick and Glaser, 1976.)

order, we can see the process by which the parallelogram problem is solved.

This somewhat gross analysis of the mental processes involved in one problem's solution provides an insight into how people solve problems in general. It also suggests some ways that we can plan a sequence of instruction to facilitate problem solving. In designing the sequence of instruction for the area module, we began with Resnick and Glaser's information-processing analysis of problem solving as a basic outline. We modified some of the steps of the analysis where tests with children (using instruction based on that outline) had suggested the analysis was incomplete, and we added some instructional activities based on Piaget's work, which we describe later in this paper.

The complete sequence of instruction in the module is shown in Table 3 above. The module first teaches the child to recognize the attributes of rectangles, figures whose areas he or she will later learn to measure. Next, the child's conception of area is developed through the exercise of comparing objects with different areas, and the method of measuring area by using unit squares is taught. Then the child is taught how to transform the shapes of objects and how to compare their area before and after they have been transformed (area conservation). Finally, the student learns some ways to transform non-rectangular objects into rectangles so that their area can be measured using the unit squares. In summary, the area module is designed to provide the student with the kinds of skills and information needed to make the appropriate decision at each step of the schematic outline for solving the parallelogram problem (Figure 5).

James Greeno (1978), another information-processing psychologist, has proposed a typology of problems that provides an enlightening contrast both to Getzels' method of classifying problems and to Wertheimer's emphasis on conceptual understanding. Greeno's typology is not based on a problem's origin or state of knowledge with respect to its solution, nor on the concepts one needs to solve a problem. It is based, rather, on the set of mental processes an individual

must possess in order to solve a particular type of problem. The typology is illustrated in Figure 6. Greeno's typology is useful because it helps to make clear a distinction between the subject matter content of a problem and the mental processes (the logical or reflective thinking) that a person must apply in solving the problem. Under each problem type are shown various problems with different content. But presumably, despite differences in content, certain groups of problems make similar demands on mental processes. Thus, as children develop the problem-solving skills necessary to solve, say, the water jug problem, we might expect them readily to solve other transformation problems like the coin and river-crossing problems.

The problems shown in Figure 6 are examples of relatively "pure" problem types in which the content and mental processes can be analyzed as distinct entities. Actually, in most real-life problems, and even in most formal academic problems, it is difficult or impossible to separate the mental processes from the content. Information-processing psychologists, however, in their efforts to develop general theories of how people solve problems, choose problems to study whose content can be isolated from the mental processes involved in their solutions.

To clarify the distinction between content and processes, let us take a careful look at one type of problem, the analogy. The analogy is usually given as a problem of this sort:

> From the list below, choose the word that completes the analogy:
>
> Hexagon: Pentagon:: Rectangle: _____
>
> (a) circle (d) prism
> (b) square (e) octagon
> (c) triangle (f) decagon

To solve this problem, a person must first be aware of the general structure of analogies, namely, that there is a relationship between

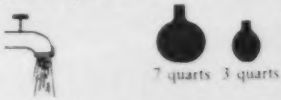
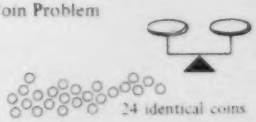
Problem Type	Prototypic Examples	
Analogies (Including Structure)	Geometry Hexagon : Pentagon : : Rectangle : _____ Social Studies Congress : United States : : _____ : Japan Science solid : melts : : liquid : _____	
Transformation	Water Jug Problem  	Measure out exactly 5 quarts of water.
	Coin Problem  	Find the one coin that's heavier than the others with fewest possible weighings.
	River Crossing Problem 	Find a way for 3 people to get across the river when they have 1 boat that will hold only 200 lbs.
Anagrams (Arrangement)	Simple Anagrams a c t	Rearrange these letters to form another word.
	Cryptarithmetic D O N A L D + G E R A L D R O B E R T	Find a number from 0 - 9 to substitute for each letter so that the addition is correct. Hint D = 3
	Geometric Puzzles 	Rearrange these pieces to form a square.

Figure 6. A problem-solving typology based on required mental processes. (Adapted from Greeno 1978.)

the first two terms and a similar or nearly similar relationship between the second two terms. (Occasionally the significant relationships occur between other pairs of terms, and the solver must establish where the relationships are.) The solver must induce (figure out) the relationship between the first two terms and look for a word in the given list that will produce a similar relationship for the second pair of terms. In the example above, the first two terms are "hexagon" (a six-sided plane figure) and "pentagon" (a five-sided plane figure). The relationship between these terms can be stated as follows: The terms both refer to plane figures, and the second term describes a figure that has one less side than the first. Is there a word in the given list that bears a similar relationship to the third term, "rectangle?" The word "triangle" completes the second pair of terms to form the similar relationship: A rectangle and a triangle are both plane figures, and the second figure has one less side than the first. If no word in the list completes the particular relationship the solver has thought of, then he or she must find another relationship between those terms and again search the list for a word that has the same relationship to the third term.

All analogy problems require a certain knowledge of content. To solve our sample analogy, a person must know the meanings of particular words from geometry (hexagon, pentagon, rectangle, circle, square, triangle, prism, octagon, decagon). To solve the social studies analogy given in Figure 6, one must know some factual information about legislative bodies in various countries; to solve the science analogy, one must know about changes in the physical state of matter. However, the solutions to all three analogies involve the same basic mental processes: (a) inducing a relationship between the first pair of terms; (b) searching (either through the given list or through memory) for a word that completes a similar relationship in the second pair of terms; and (c) determining if the problem has been solved (if not, the sequence is repeated).

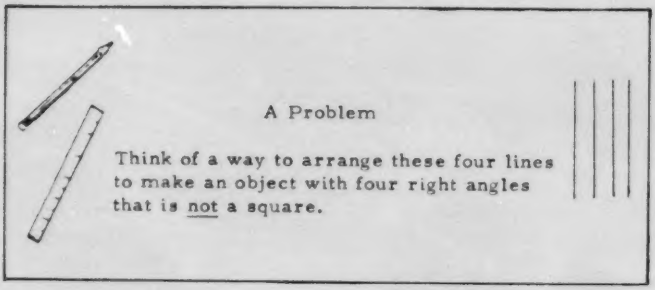
Similarly, there are common mental processes in the solutions of transformation type problems. All three transformation problems shown in Figure 6 specify an initial set of conditions and a desired

outcome. The solver must infer which manipulations of the initial conditions are possible and then plan out a series of these manipulations that will lead to the desired outcome.

In the arrangement type of problems, although each of the examples shown requires some content knowledge for its solution, all present the solver with an initial arrangement of certain elements and specify a certain rearrangement of these elements as the desired outcome. The solver must construct the several possible rearrangements of the given elements in turn and select the one which was specified as the outcome.

Greeno's typology draws our attention to the possibility of explicitly teaching processing skills, as distinct from conceptual knowledge. An interesting theoretical and instructional issue might be how instructional time and effort may be optimally balanced between teaching content and teaching processing skills. We leave it as an exercise for the reader to determine the balance we struck in the area module, a balance largely determined by intuition and our experience.

We believe that certain problems in the area module help students develop general processing skills that are applicable to a variety of problem-solving situations. For example, at the end of a lesson on squares and rectangles the student is presented the following problem:



A mental process necessary to solving this problem is recentering, a Gestalt term meaning seeing the problem in a whole new light. That is, the student who has just been working with "closed" figures, such as squares and rectangles, must recognize that the goal of the problem is not necessarily to draw a closed figure. He or she must imagine other kinds of figures that will fit the demands of the problem. Recentering is an example of a mental process applicable to numerous problem situations regardless of their content.

A Structuralist View

The distinction between content and mental operations in problem solving is not absolute. In fact, some psychologists believe that when a child is unable to understand a certain concept, which in this context means to be able to use a concept to solve problems, it is because the mental operations necessary for understanding are not available to the child. The interactive relationships between content and mental operations in problem solving are represented in the work of Jean Piaget, the Swiss psychologist, who for nearly half a century has engaged in research to broaden our understanding of the question, "How does knowledge develop?" To answer this question, Piaget has studied the development in humans of the ability to reason and to incorporate a system of logic into their thinking.

Piaget assesses an individual's ability to reason by presenting the individual with problems to solve. For the infant, a typical problem might be to find a toy hidden under a pillow. An eight-year-old might be asked to compare the areas of a square and a rectangle whose perimeters are equal. A problem appropriate for a teenager might be to predict the level of water in the left side of a U-tube when water is added to the right side. Piaget and his co-workers have confronted hundreds of children with these and other problems drawn from science and mathematics. The responses children give to the problems and the reasons they give for their responses are carefully analyzed to determine the underlying structure of their thinking. To the extent that a child's answers seem to follow a logical pattern, Piaget assumes

a certain organization of operations. Hence, the Piagetian problems serve as probes into the mental operations.

Among the problems Jean Piaget and his colleagues have studied are various problems about area (Piaget & Inhelder, 1967; Piaget, Inhelder, & Szeminska, 1960). They have developed a series of problems (Figures 7 through 9) that can be used to probe children's mental operations and to define operationally what is meant by the phrase, "understand the concept of area." The first problem in the series probes the child's understanding of an axiom from Euclidian geometry: If equals are subtracted from equals, the results are equal. The problem in adult terms, the problem as presented to the child, and answers children usually give to the problem are shown in Figure 7.

A second set of problems tests the child's understanding of the fact that, under certain conditions, the shape of an object can be transformed while its area remains the same. This is an important concept for solving the parallelogram problem--i.e., one may transform the parallelogram into a rectangle that has the same area. (Until the principle of area conservation is understood, children are unable to make the judgment of area equivalence required in Figure 5, Step E--"Is the area of R the same as the area of P?") The transformation problems in adult terms, the problems as presented to the child, and the responses children usually give to the problems are shown in Figure 8.

The next step in Piaget's analysis of the child's understanding of area is to present the child with the problem of measuring the areas of some figure by the method of <u>superposition</u>, which involves simply placing one figure on top of another and comparing their sizes. The problem in adult terms, the problem as presented to the child, and the answers children usually give to the problem are shown in Figure 9.

It should not surprise you that most children eight years old and younger do not understand the Figure 9 problem--that is to say, they cannot solve it. You may have wondered why Piaget added the complication of asking children to compare the measurements of objects having different shapes. One reason was to test whether or not the children were able to recognize that two objects that are equal in area

The problem in adult terms:

If A = B
and C = D
then A - C = B - D.

If the area of A is equal to the area of B,

and the area of C is equal to the area of D,

then the area of (A - C) is equal to the area of (B - D).

Figure 7. A Piagetian problem on subtracting equal areas from equal areas.

–continued–

Farmer Brown's Field

Farmer Jones' Field

The child is shown two green boards (which represent fields) and two toy cows, and shown that the fields are "the same size" and that both cows have "the same amount" of grass to eat.

Farmer Brown decides to build a barn on his field. The barn covers some of the grass.

The child is asked, "Do both cows now have the same amount of grass to eat?"

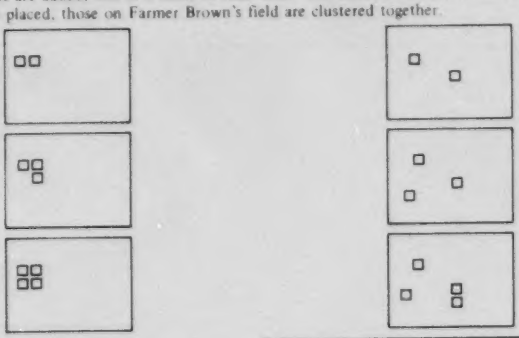
Here the child generally states that Farmer Brown's cow has less grass to eat than Farmer Jones' cow.

Farmer Jones decides to build a barn on his field, too.

The child is shown another barn, identical to the first (the barns are placed in corresponding locations on the two fields), and compares its base with the base of the first barn. The child is asked, "Do both cows now have the same amount of grass to eat?"

Here most children agree that both cows have the same amount of grass to eat.

Now barns are added, one at a time, to each field. The barns on Farmer Jones' field are randomly placed, those on Farmer Brown's field are clustered together.



The child is again asked whether both cows have the same amount of grass to eat.

Here children younger than seven usually answer that Farmer Brown's cow has more grass to eat.

Figure 7 (continued).

The problem in adult terms:

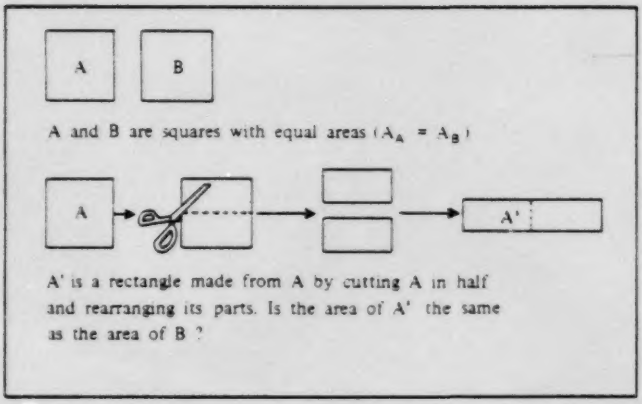
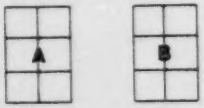

A
B
A and B are squares with equal areas ($A_A = A_B$)
A
A'
A' is a rectangle made from A by cutting A in half and rearranging its parts. Is the area of A' the same as the area of B ?

Figure 8. A Piagetian problem on area as a stable attribute of an object.

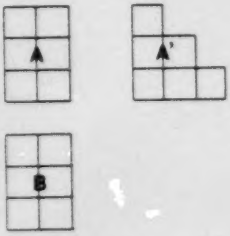
–continued–

The problem as given to the child.



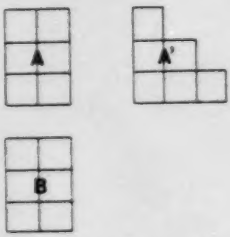
The child is shown two rectangles, each made from six squares.

Children readily agree that rectangles have the "same" amount of room on top (area).



A square from one of the rectangles is moved to form a figure of another shape. (A').

Children younger than seven years generally assert that one or another of the figures has more or less area ("room on top of it").



In another version of the problem, rectangle B is covered with small squares and the child is asked, "How many squares would it take to cover A' (more, less or the same number it takes to cover B)?"

It is interesting to note that some children say it takes the same number of squares to cover A' as to cover B, but then say the area of B is different from the area of A'.

Figure 8 (continued).

The problem in adult terms.

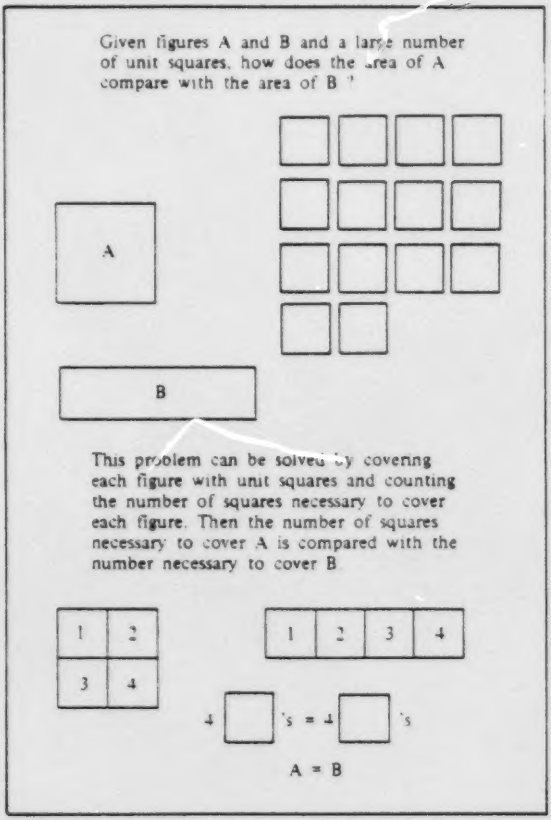
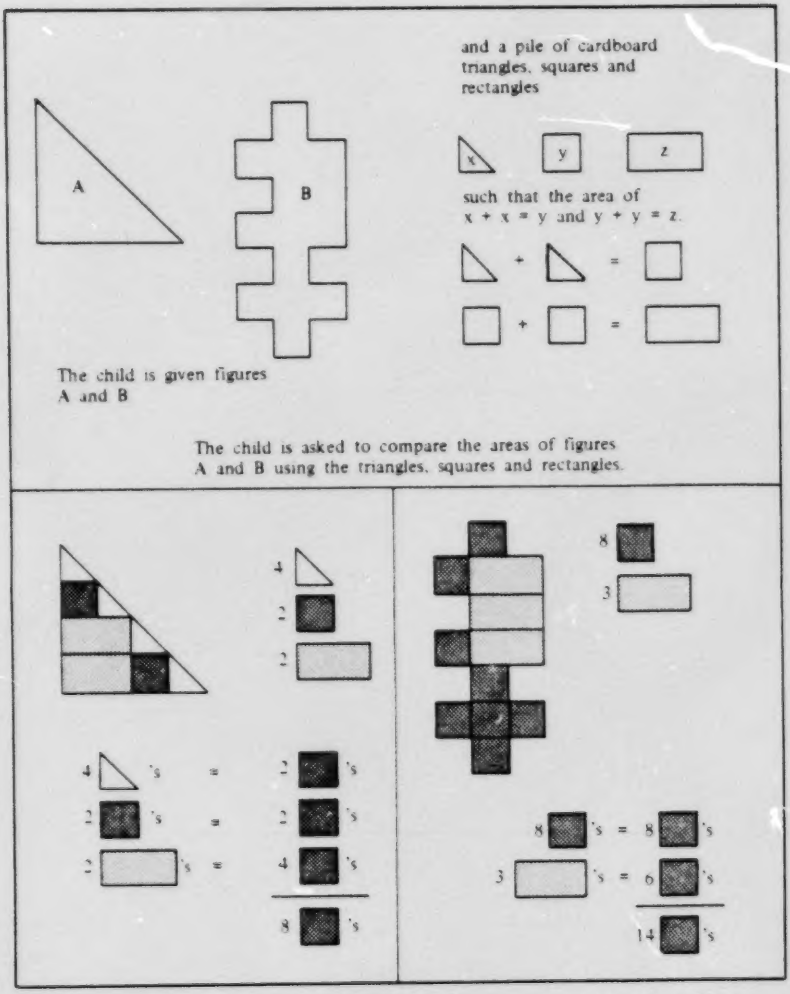

Given figures A and B and a large number of unit squares, how does the area of A compare with the area of B ?
A
B
This problem can be solved by covering each figure with unit squares and counting the number of squares necessary to cover each figure. Then the number of squares necessary to cover A is compared with the number necessary to cover B.
1
2
3
4
1
2
3
4
4
's = 4
's
A = B

Figure 9. A Piagetian problem on transitivity in area measurement.

–continued–

The problem as presented to the child:


A
B
The child is given figures A and B
and a pile of cardboard triangles, squares and rectangles
x
y
z
such that the area of x + x = y and y + y = z.
The child is asked to compare the areas of figures A and B using the triangles, squares and rectangles.
4
2
2
4 's = 2 's
2 's = 2 's
2 's = 4 's
8 's
8
3
8 's = 8 's
3 's = 6 's
14 's

Note: There are, of course, other combinations of shapes that will cover A and B

Figure 9 (continued).

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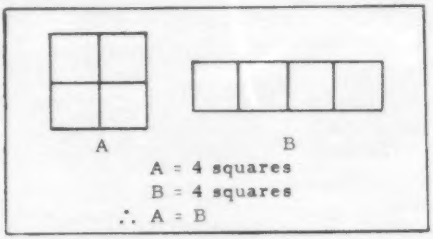
to the same object are equal in area to each other--the transitivity relation (i.e., if A = B and B = C, then A = C).

An adult doing this problem would probably reason in this way:



While this seems simple to adults, it is not obvious at all to young children. Only when the child is confronted with a problem of the ingenious type that Piaget created do we become aware that the child's thinking is not organized in a way that allows him or her to grasp transitivity. Only when transitivity is understood by the child can the child correctly use a measuring device (a unit square or a ruler) to compare the areas or lengths of two objects.

If you have ever watched a child trying to use a ruler to measure the length of an object that is longer than the ruler, you are probably aware that the child is confronted with another difficulty in making measurements. The nature of the difficulty can be inferred from the haphazard way the child places a ruler for successive measurements along a line. For one thing, in placing the ruler along the edge the second time, the child does not place the left edge of the ruler at the point on the object where the right edge was. The child's placement of the ruler along the object is not coordinated in space.

The use of a 1-foot ruler to measure an object longer than one foot is an example of measurement of successive iterations. Measurement by successive iterations, whether of length or of area, is different from and more complex than measurement by superposition, which was illustrated in Figure 9. Piaget tests the child's ability to

measure area by successive iterations by asking the child to find the area of a plane geometric figure using a single unit square. Since only a single unit measure is avilable to the child, he or she must make several successive placements of the unit square, coordinating each placement in space so that there is no overlap or gap. To carry out this process successfully, the child must impose some mental organization on the surface of the figure, e.g., visualizing it as a grid composed of identical squares. This implies a certain level of structure in the child's mental operations.

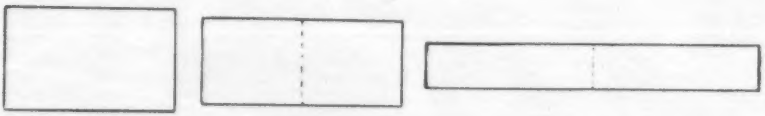
Each of the Piagetian tasks or problems we have described here highlights some mental operations involved in a child's understanding of the area concept.[3] According to Piaget, the physical manipulations the child carries out with objects are analogous to the mental operations necessary for understanding the concept of area. By extension, providing children with concrete experiences such as Piaget describes may help them develop the mental operations needed to understand area. At the least, analyzing the responses a child gives to the tasks can help you judge whether his or her thinking includes these mental operations. If these operations are not available to a child, it is not surprising that he or she is unable to demonstrate an understanding of area even after intensive instruction.

In the area module we have included several problems similar to those of Piaget. For example, in one problem, the student is given three congruent rectangles, as shown below:

A B C

[3]There are alternative theoretical explanations for the observations of children's behavior when confronted with these Piagetian tasks. The differences in explanation, while important, should not detract from the value of the tasks as probes into childrens' thought processes. The insights into children's learning difficulties that can be gained from observing students as they confront these problems are striking.

After cutting the rectangles B and C apart along the dotted lines, the student is asked to rearrange the pieces to form these figures:



A B C

Then the student is asked to measure the area of each rectangle with unit squares and to compare the areas of the three rectangles. It has been our experience that some children who can accurately measure the area nevertheless state that the area of rectangle B is less than the area of rectangle A. Thus, it should not be surprising to find that some students who have worked through the area module still do not fully understand area and its measurement.

With the above discussion, we probably have succeeded in demonstrating to you that area is a more complex concept for children to learn than you had previously thought. You should no longer be surprised that children confuse area and perimeter and often cannot decide which formula to use. Before teaching students how to calculate area (or perimeter), a teacher must be sure they know what area is. The Piagetian tasks we have described can be used to diagnose whether or not a student understands area. If he or she does not, the learning experiences provided in our area module should help remedy the lack of understanding. When students come to understand area, they will not only be able to do the usual academic problems in their math books, but may also be able to do Type 2 problems like the parallelogram problem.

Applying Problem-Solving Theory to Classroom Instruction

We hope the ideas we have presented here will persuade teachers seriously to consider including some problem solving in their

teaching and to try our problem-solving approach to teaching the concept of area. We realize that all teachers have an enormous number of activities to carry out in a limited amount of time. Designing new learning experiences for students and dealing with the innumerable details that are part of creating instructional materials are certainly time-consuming undertakings. To help ease the difficulties and lessen the time demands of introducing problem-solving activities into the classroom, we have written a largely self-instructional module on the concept of area that you can use to add problem solving to your current teaching methods.

The area module includes a number of self-instructional lessons and small group sessions that provide learning experiences on the concept of area and its measurement via a problem-solving approach. The module is designed so that the child does not learn about area only in the abstract, but also learns to understand the concept through actively manipulating materials. Looking through the lessons of the module, you will note specific manipulative experiences that are essential for developing children's understanding of area and its measurement, including experiences to develop understanding of prerequisite concepts such as length, square, and rectangle. If you compare the problems at the ends of the various lessons with the instruction that precedes them, you will become aware of the appropriateness of the problem tasks that have been selected for each point in the instruction.

Our hope is that you will use this module with your students, comparing the understanding they achieve by means of the problem-solving approach with the understanding they achieve through traditional approaches that rely on practice of standard algorithms. If this paper has fulfilled its purpose, we have convinced you that a problem-solving method will both help you teach the concept of area and help your students understand it. Further, we want to have given you, in the area module, a way to take some time out of problem solving with your students.

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